Joe Bottini

May 29, 2018

*Nuclear Reactor Physics*

Weston M. Stacey

Second Edition

Wiley-VCH

The binomial theorem is given,



where



for any non-negative integer .

Consider the proof by induction. First, prove the base cases of  and .

For ,





For ,







Suppose



It must be proven that



Consider ,



Substitute Eq (8) into Eq (10),



Distribute the sum ,



Simplify the first sum and change the index on the second sum,



Simplify the second sum,



Combine the sums and consider the end terms,



Need three identities to complete the inductive step.

First, consider the coefficient in the sum,



Factor similar terms,



Make a common fraction,



Rearrange terms,



Simplify the factorials,



Finally, substitute Eq (2),



This is Pascal’s Identity.

Next, consider the following cases,









Equations (23) and (25) hold for all . Therefore, it follows that,



and



Finally, substitute Eqs (21), (26), and (27) into Eq (15),



Extend the limits on the sum by combining the last two terms,



QED